The regularity of cycles in permutation groups

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1 Content

Since the end of previous centurythere has been a growing interest in applicationof probability in finite groups. Probability can be applied to the theory of finite groups in a number of cases like probabilistic statements about groups, construction of randomized algorithms in computational group theory or application of probabilistic methods to prove deterministic theorems in group theory. When we talk about probabilistic statements about groups, we talk about thestatistical group theory.
We are interested in the number of regular cycles in a given permutation group of degree n. In more detail, let$G$be a subgroup of S_n. Thenwe have a conjecture, that

$$\frac{|C(G)|}{|G|} \leq \frac{\varphi(n)}{n},$$

where$|C(G)|$ is a cardinality of the set of all cycles of length n, $\varphi(n)$ is*Euler's totient function.*

This problem also can be interpreted asthe probability, that a randomly selected element from permutation group will be a cycle of length n(generates a transitive group).Another interpretation of this problem is related to the notion ofthe cycle index. See thebook *Graphical enumeration* Frank Harary, Edgar M. Palmerfor further details.

Moreover it should be mentioned thatGareth A. Jones referred in his article***Primitive** permutation groups containing a cycle*on similarly questions, which have been raised by Zvonkin.

The statistics of random permutation,such as the cycle structure are of fundamental importance in the analysis of algorithms, especially of sorting algorithms, which operate on random permutations.

2 The purpose of the investigation

There are three goals of the investigation:

a. Prove the conjecture set, specificallythatfor all subgroups Gin S_n an inequality

$$\frac{|C(G)|}{|G|} \leq \frac{\varphi(n)}{n},$$

holds.

b. Find all subgroups for which equality is achieved.

c. Improve the estimate for some specific classes.

3 Methods of the investigation (in comparison with known methods)

The methods used are combinatorial andprobabilistic methods in the theoryfinite groups.

We also used the computer algebra system GAP. We developed an algorithm to successfullyverify the conjecture for groups of degree 16 – 30. GAP was chosen, because it has a classification of all transitive subgroups of the symmetrical group up to conjugacy of order less than 31.

4 Results

In this work the hypothesis was proved for primitive and imprimitive groups separately.

A partition of theset$\{1, \dots, n\}$is

$$\{1, \dots, n\} = \bigcup_{i=1}^{m} B_i,$$

where$\{B_i\}_{i=1}^{m}$ are mutually disjoint sets. Primitive groups are groups, which do not preserveany nontrivial partition; otherwise groups arethe group is called imprimitive.

The case of primitive groups is based on the Jones'sclassification, which was published in 2004. He listed all primitive groups that contain at least one cycle of length n. But nothing about the number of such cycles was known.

We prove theconjecture for solvable primitive groups. For unsolvable primitive groups we improve an upper bound.

Theorem 1. Let G be an unsolvable primitive permitation group of degree n. Then

$$\frac{|C(G)|}{|G|} \leq \frac{2}{n}.$$

Moreover, the equality holds if and only if G is A_n for odd $n \geq 3$, $P\Gamma L_2(8) \subset S_9$, $PSL_2(11) \subset S_{11}$, $M_{11} \subset S_{11}$, $M_{23} \subset S_{23}$.

The case of imprimitive groupswas reduced to studyingsuch interesting structuresasthe wreath products of groups.

Wreath product of groups G and H $(GwrH)$can be constructed in the next way. Take m copies of group G

and let group Hacts on these$m$copies. Other words, wreath productof groups G and H is a semidirect product of a direct product of m groups G with group H.

Theorem 2. Suppose the conjecture holds for subgroups $G \subset S_n$and $H \subset S_m$. Then it also holds for the wreath product $G\ wr\ H \subset S_{nm}$.

Theorem 3. The conjecture holds for arbitrary subgroups of the wreath product of two cyclic groups $C_n\ wr\ C_m \subset S_{nm}$.

In addition, we explicitly describe all cases from *Theorem 3*, when the equality holds. This involves various techniques from number theory.

5 Conclusion

Summing upwe obtained the following new results.The frequency of the cycles of the length $\boldsymbol{n}$ in permutations groups of degree $\boldsymbol{n}$was estimated. Forunsolvable primitive groups we obtained an exact upper bound of $\frac{\mathbf{2}}{\boldsymbol{n}}$ and described all cases when this bound is achieved. For imprimitive the estimate wasknown only trivial cases likewrest product of two cyclic groups. This paper presents the results for wreath product of arbitrary groups and for all subgroups of wreath product of cyclic groups.

In the latter case we found call such subgroups,when equality holds.

Triangle-free graphs

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Graph theory today is a quickly developing section of discrete mathematics. Graphs are used in communicational schemes, electronic and electric circuits and other spheres.

Triangle-free graphs are mentioned in the article *Size in maximal triangle-free graphs and minimal graphs of diameter 2* by C. Barefoot, K. Casey, D. Fisher, K. Fraughnaugh and F. Harary. A triangle-free graph is called maximal if the addition of any edge creates a triangle. For $n \geq 5$ was shown that there is an n-node m-edge maximal triangle-free graph if and only if $m = k(n - k)$ or $2n - 5 \leq m \leq \left[\frac{(n-1)^2}{4}\right] + 1$. The number of such graphs, their structure also were being explored (*The number of maximal triangle free graphs* by J. Balogh and S. Petrickova, *The typical structure of maximal triangle-free graphs* by J. Balogh, H. Liu, M. Sharifzadeh and S. Petrickova)

By analogy, let the graph which doesn't contain complete p-node subgraphs be called maximal p-free, if the addition of any edge creates complete p-node subgraphs. We show that,for $n \geq p + 2$, n-node m-edge maximal p-free graphs exists if it is a complete $(p–1)$-partite graph or $(p-1)n - \frac{p(p-1)}{2} - 2 \leq m \leq \left[\frac{(n+2-p)^2}{4}\right] + 1 + \frac{(p-3)(2n-2-p)}{2}$. Also noted that every complete $(p–1)$-partite graph with at least one part more than 1 is a maximal p-free graph.Though, we don't know, are there any maximal p-free which neither satisfy the inequality, nor $(p–1)$-partite.

The girth of graph is the length of a shortest cycle contained in the graph. We show that the girth of maximal triangle-free graphs can be 4, 5 or ∞ (no cycle) and the girth of maximal p-free graph for $p>3$ is 3 for $n \geq p$.Also noted that if the independent domination number of a maximal triangle-free graph less than its minimum degree, then the girth of the graph can't be 5.

Was shown, that the minimum degree δ of n-node maximal p-free graph is at most $\left[\frac{n(p-2)}{p-1}\right]$.

Pythagorean Triple Siblings, with Conjugate Legs

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6 Main Goal

Main purpose of our research is investigation of the following Diophantine system:

$$\begin{cases} c^2 - (p^2 + q^2)^2 = \square \\ c^2 - (p^2 - q^2)^2 = \square \end{cases} \qquad (1)$$

Firstly, we try to find solutions by using computer and we check

$$c \leq 650.$$

Unfortunately, computer search not give us non-trivial solutions so, we thought the following hypotheses is true: this system has no non-trivial solutions.

But, from the history of number theory we know a distinguished example about Euler quartic conjecture, which was thought to be true for more than two centuries.

In 1772, Euler proposed that the equation

$$A^4 + B^4 + C^4 = D^4$$

had no solutions in integers [1] .

This assertion is known as the Euler quartic conjecture. Ward [2] in 1948 showed there were no solutions for

$$D \leq 10\,000.$$

This was subsequently improved [1] to:

$$D \leq 220\,000.$$

However, the Euler quartic conjecture was disproved in 1987 by N. Elkies, who, using a geometric construction, found:

$$2682440^4 + 15365639^4 + 18796760^4 = 20615673^4$$

In 1988, Roger Frye found

$$95\,800^4 + 217\,519^4 + 414\,560^4 = 422\,481^4$$

and proved that there are no solutions in smaller integers and showed that infinitely many solutions existed.

What about our system? Of course it's possible for the system to have non-trivial solutions. But similarly to Elkies case, solutions, if they exist, must be in large numbers.

Our research is dedicated to proving the following theorems.

Theorem 1:

Diophantine system (1) has non-trivial solutions.

Theorem 2:

There are infinitely many non-trivial solutions of (1).

We obtain general parametric formulas for solutions of our Diophantine system.

7 Research

We use a theorem about rational solutions of Kummer's surface of special type [4]:

All nontrivial rational solutions of the following equation:

$$\mu^2 = \alpha\beta(1-\alpha^2)(1-\beta^2)$$

are given:

$$(\alpha, \beta, \mu) = \left(\frac{X}{N}, \frac{Z}{N}, \frac{YW}{N^3}\right)$$

where (X,Y) and (Z,W) are different rational nontrivial solutions of C_N congruent curve equation. i.e.

$$Y^2 = X(X^2 - N^2), \; W^2 = Z(Z^2 - N^2).$$

Nontrivial integer solutions of system

$$\begin{cases} c^2 - (p^2 + q^2)^2 = \square \\ c^2 - (p^2 - q^2)^2 = \square \end{cases}$$

are given by formulas:

$$c = k\sqrt{xz}(x + z)(xz + N^2)$$
$$p^2 = kxz(x + N)(z + N)$$
$$q^2 = kxz(x - N)(z - N)$$

Where, N is a congruent number, x and z are different non-trivial solutions with squared product to a congruent equation.

$$Y^2 = X^3 - N^2X$$

Therefore, by substituting k, x, z and N with numbers, we get the following numerical examples. Solutions of the corresponding congruent equation are taken from [4].

8 Numerical examples:

1) N=6; x=18; $z = \frac{19602}{47^2}$

$$15358381995^2 - (114774^2 + 35673^2)^2 = 5215702800^2$$

$$15358381995^2 - (114774^2 - 35673^2)^2 = 9708645804^2.$$

2) N=34; x=162; $z = \frac{2178}{7^2}$

$$3322469535^2 - (50127^2 + 14784^2)^2 = 1891797600^2$$

$$3322469535^2 - (50127^2 - 14784^2)^2 = 2403264864^2.$$

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Infinite flow of random numbers and its possible uses

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INTRODUCTION

Have you ever decoded any kind of an anagram or a puzzle, where bychanging the order of the lettersin a word, you get the different one (word)? Have you ever bought anything or checked your bank account via internet? If Yes, then you may have entered the world of codes, passwords, encrypted and decoded information. People change information by a message, for what they use different modes starting with direct contact and ending with complex equipment. The part of information is confidential and needs to be protected. Protecting the information means the costs, which a person gives for coding or decoding. Once there was a time, when only government, ambassadors, mysterious or military secret services used the encoder machines. But today, everything has changed. With the invention of computer and internet, protecting information can be done via different means, including passwords, which a consumer has to submit/log in every time before getting the needed information. But the password is information itself, which also needs to be protected. According to these, keeping the information safe has not been as vital/urgent as it is now.

Correspondingly, we are interested in how safe our private information is and how we canreinforce the security measures with minimum amount of charges.

THE PURPOSE AND METHOD OF THE INVESTIGATION

The aim/goal of the paper is to come up with an easily usable and inexpensive method of defending short electric messages. Our offered idea of covering

a message is an original usage of flow of random numbers. It grants an algorithm a specifics of ‘’unitary notebook’’. A source of random numbers flow is the machine itself which sends the message (this could be a mobile phone), thus we don’t need to make the special generator. In the process of encryption, we use text tabulation, classical movable crypt (Symbols move in a text) and Interchangeable one (One symbol or a group of symbols change with others in a text).Introduced terms and transformations are fully described in verbal way as well as analytical way. A usage of the specific sample is also represented.

CONTENT

Let's say, one customer is sending short electric message (SMS) to another person. Which numbers can this procedure be described with? This kind of numbers are, for example, the phone number of both - sender and receiver; mobile operator settings (Maximum quantity of symbols); quantity of symbols; date of sending (day, month, year); time/date of sending (hours, minutes, seconds). It's interesting that all these features never repeat and we may face them only once at the sorting. Also, It's impossible to consider the calculation for every sorting - To calculate it, we have to check every possible outcome. Let's call this described unity of numbers a "Initial Random Settings" (IRS). The method we offer for the privacy of messages is based exactly on IRS.

Every message so text is a strictly organized sequence of alphabets and punctuation marks. The first step of encryption is the replacement of alphabets and punctuation marks to double-digit codes, which is implemented according to the special table (this table is according to IRS) - We get a sorted sequence of two-digit codes. The next step is thetabulationso the division of a square matrix type subgroups, the size of it is also depending on IRS. The third step is a reversible conversion of tables, which is done/implemented by a randomly selected (Based on IRS) special operators of a set of pre-defined "instruments".

Let's say, Phone operator setting is K, also - the first step of SMS encryption is done and The text containing N symbol is already encrypted to a sorted sequence of two-digit codes, then

$$z = \left[\sqrt{\frac{min\{N; K\}}{2}}\right]$$

We called z a reasonable minimum of text-table (The minimum size of text-table is z x z). We chose n x n for a size of text-table, where n is possible smallest natural number, for which

$$n - z \equiv ((a_1 + 1)a_2 + (a_3 + 1)) mod(z + 1)$$

Here and after herea_1 is/will be the selected settings from IRS. It's clear that text-table size varies from z to 2z. n defines a rule according to which, there is excreted four movable zone (quarter) in a table, and unmovable elements in private case.

Set of "Instruments" is described in the method - this is a unity of reversible transformations of table quarters, including axial symmetry, horizontal or vertical dislocation. In every specific case, it's defined from IRS which instruments we use for objects. Every instrument is described in a analytical way, for example, one of the instruments is a dislocation of first quarter line elements ("Horizontal conveyer"). If n is even, we establish designation$t = \frac{n}{2}$and find the smallest possible k natural number, for which$\equiv a_4 (modt)$, then dislocationf_k^I of$i$-line elements$(A_{ij},\ j = \overline{1, t})$ is defined by the formula :$f_k^I : A_{i\ t+j} \rightarrow A_{i\ t+p}$, where p is the smallest possible natural number, for which $p \equiv j + k\ (modt)$, and i=1,t. The same operator for odd n is defined so:

$$f_k^I : A_{i\ t+1+j} \rightarrow A_{i\ t+1+p}, \qquad \text{where } t = \left[\frac{n}{2}\right]$$

There is an illustration of method in the work: The sample text got encrypted at first and then opened by the reversible operator.

CONCLUSION

Presented method in the paper is structured based on the level of an algorithm, so is practically implementable and in the case of phone operator ability, it can provide people confidentiality of the private conversation. The method is easy and inexpensive, which doesn't need serious technical items. At the same time, breaking the crypt requires a lot of time and resources. Even if the crypt is broken result are disposable, since they can't be reused because of random numbers taking part in algorithm.

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Diophantine Equation Involving Four Biquadrates and a Square

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10 Main Goal

The main goal of our research is investigation of a Diophantine biquadrate equation:

$$p^4 + q^4 - a^2 = r^4 + s^4 \quad (1)$$

under an additional condition:

$$pq = rs$$

All of humanity's knowledge regarding Diophantine equations is gathered in the books: "History of the Theory of Numbers, Volume II: Diophantine Analysis" by Leonard Eugene Dickson [1] and "An Introduction to Diophantine Equations" by Titu Andrescu, Dorin Andrica, Ion Cucurezeanu [2]. Unfortunately though, these books do not contain any information concerning our type of 4th degree equation. Therefore, our research is original and innovative.

11 Method

To research our equation, we have to use rational solutions of the congruent curve equation

$$C_N : y^2 = x(x^2 - N^2)$$

By using solutions of the congruent number equation C_N, it is possible to construct a three-term arithmetic progression of squares with N difference [3]. In our research, these solutions are used to find parametric solutions of the above-mentioned Diophantine system.

12 Research

Actually, our equation is equivalent to a Diophantine system:

$$\begin{cases} (p^2 + q^2)^2 - a^2 = (r^2 + s^2)^2 \\ (p^2 - q^2)^2 - a^2 = (r^2 - s^2)^2 \end{cases} \quad (2)$$

because by adding up the equations in (2), we get equation (1) and by subtracting them we get the additional condition.

So, it's clear, that once we find a solution to (2) it will be a solution to (1) as well.

We begin to research the Diophantine system (2). It is possible to divide the system into two subsystems:

$$\begin{cases} (p^2 + q^2)^2 - a^2 = \square_1 \\ (p^2 - q^2)^2 - a^2 = \square_2 \end{cases} \quad (3)$$

$$\begin{cases} a^2 + (r^2 + s^2)^2 = \square_3 \\ a^2 + (r^2 - s^2)^2 = \square_4 \end{cases} \quad (4)$$

We use a theorem about rational solutions of Kummer's surface of special type [4]:

All nontrivial rational solutions of the following equation:

$$\mu^2 = \alpha\beta(1 - \alpha^2)(1 - \beta^2)$$

are given:

$$(\alpha, \beta, \mu) = \left(\frac{X}{N}, \frac{Z}{N}, \frac{YW}{N^3}\right)$$

where (X,Y) and (Z,W) are different rational nontrivial solutions of C_N congruent curve equation. i.e.

$$Y^2 = X(X^2 - N^2), \; W^2 = Z(Z^2 - N^2).$$

13 Solutions of (3)

We get, that all integer solutions of (3) can be given using these parametric formulas:

$$a = k_1 * 4N_1\sqrt{x_1 z_1}$$

$$p^2 = k_1 * (x_1 + N_1)(z_1 + N_1)$$

$$q^2 = k_1 * (x_1 - N_1)(z_1 - N_1)$$

14 Solutions of (4)

Similarly to (3), we get that all integer solutions of (4) are given using the following formulas:

$$a = k_2 * 4N_2\sqrt{x_2 z_2}$$
$$r^2 = k_2 * (x_2 + N_2)(z_2 - N_2)$$
$$s^2 = k_2 * (x_2 - N_2)(z_2 + N_2)$$

By uniting the solutions of (3) and (4) we get the parametric formulas for solutions of the original system.

15 Results of the research

Theorem:

Nontrivial integer solutions to (1) are given by formulas:

$$a = k * 4N\sqrt{xz}$$
$$p^2 = k * (x + N)(z + N)$$
$$q^2 = k * (x - N)(z - N)$$
$$r^2 = k * (x + N)(z - N)$$
$$s^2 = k * (x - N)(z + N)$$

Where k is an integer, while x and z are nontrivial different rational solutions of an arbitrary congruent curve. It must be noted, that, k, x and z must be chosen so that p, q, a, r and s are integers.

Therefore, by substituting k, x, z and N with numbers, we get the following numerical examples. Solutions of the corresponding congruent equation are taken from [3].

16 Numerical cases:

1) $N = 5,\ x = \left(\frac{5}{2}\right)^2,\ z = \left(\frac{41}{12}\right)^2$

$$735^4 + 155^4 - 492000^2 = 465^4 + 245^4;$$

$$735 * 155 = 465 * 245.$$

2) $N = 6,\ x = \left(\frac{5}{2}\right)^2,\ z = \left(\frac{1201}{140}\right)^2$

$$8743^4 + 1151^4 - 40353600^2 =$$
$$= 8057^4 + 1249^4;$$

$$8743 * 1151 = 8057 * 1249.$$

17 References:

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ICYS_2016 abstract India (Mathematics - Monumental Geometry)
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The Purpose of the Investigation

India has a rich history and the historical monuments of India have always been admired by everyone. From Mughal to modern period, monuments have attracted people because of their aesthetically pleasing designs. Geometry has played a very important role. Architecture, Mathematics and geometry all together have resulted in creations of buildings that are both inspiring and beautiful. Hence our research on monumental geometry.

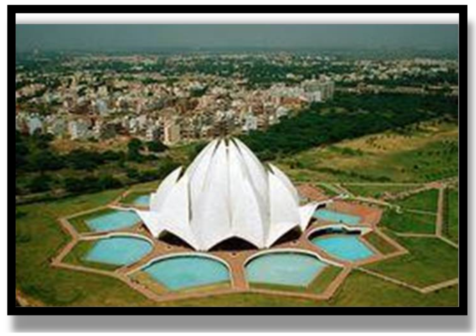
Method of the investigation

After observing various monuments, we tried to understand the geometry and mathematical concepts involved in the designs of these structures. We built models using geometrical concepts which helped in investigating the designs of the following monuments.

LOTUS TEMPLE

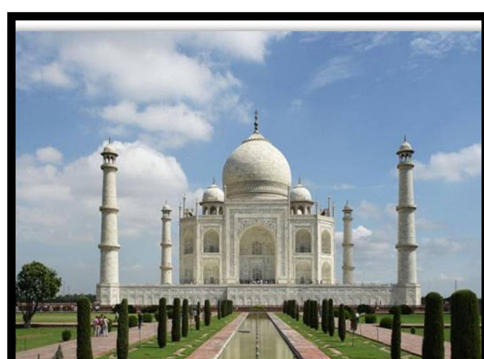
The Lotus Temple, was designed by the architect Fariborz sahba as a worship place for the Baha'i community.

A few months ago, we visited Lotus Temple,Its geometry amazed us.The temple is like a lotus that looks afloat on water. The lotus has nine components in all. Ponds, arches and all the petals are nine in number. The structure is open from the top and covered with a glass which illuminates the whole dome. It has got an eco- friendly environment which complements the architecture of the superstructure.



TAJ MAHAL

The Taj Mahal was built by the great ruler Shah Jahan, in the memory of his wife Mumtaz Mahal. One of the Seven Wonders of the World, whose symmetry and harmonious architecture compliment its beauty, the Taj Mahal was built in the 17 century. The concept of its architecture involves symmetrical designs. The minarets, rooms, walls, and even gardens follow perfect symmetry. The architecture is a tribute to mirror symmetry. Even the reflecting pool in front of the structure, which reflects the beautiful image of the entire monument ,has a lot of symmetry. This structure is a perfect example of blissful architecture.



EIFFEL TOWER

This great structure was built under the supervision of the architect Gustave Eiffel.

One of the most perfect examples of amazing European architecture is the Eiffel tower. The basic structure of the tower is lattice-work columns at each of the four corners of the Tower, in which diagonals connect four elements that help in making stiff, lightweight columns.

The tower reveals an exponential shape, where the lower section is specially designed to ensure resistance to wind forces.

Results of the Experiment

The investigation revealed the various mathematical and geometrical concepts that are involved in the making of buildings.

All the monuments have their own unique designs and harmonious patterns, be it European or Indian architecture.

Conclusion

Geometry plays a very important role in architecture.

We are able to recognize mathematics, architecture and geometry with an abundant essence of history all together in the monuments found in the world. Hence, geometry can result in creation of beautiful and pleasing designs, that too with a sense of uniqueness and harmony.

Reference

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SOLUTION TO TRAVELLING SALESMAN PROBLEM (TSP) FOR LOGISTICS DISTRIBUTION IN PT JALUR NUGRAHA EKAKURIR (JNE) REGIONAL DEPOK USING THE APPLICATION OF LINEAR PROGRAMMING AND *SOLVER* PROGRAM

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1. Research purpose

One of the problems happening in logistic companies, especially PT JNE, is that the vehicles that deliver packages to several addresses and return back to the starting depot take such a long distance and time because they do not know and choose the shortest path. This can waste such amount of money, customer satisfaction, and competitiveness of the company.

The problem above can be formulated as a Travelling Salesman Problem, which can be solved quite easily because it is can be just simply formulated in linear programming and can be solved using executor engines which use algorithms or heuristics. One example is Premium Solver Platform in MS Excel, which is simple and easy to use, so it can be applicable in R&D office environment, without requiring one to learn difficult programming concepts.

With my background knowledge in mathematics and MS Excel, I want to solve this problem so JNE can perform better in the future and become able to compete with its other rivals.

2. Research method

In this research, I used linear programming which is formulated in MS Excel and then executed it using Gurobi Solver Engine in Premium Solver Platform.

Firstly, data of the 55 addresses were obtained. Then, the coordinates of those addresses were searched in Google Maps and a distance matrix between those 55 coordinates were created using Google Maps API. Next, the linear programming formulation was made in MS Excel. Finally, it was executed in Premium Solver Platform using LP Gurobi Solver Engine.

This engine, which uses branch-and-bound heuristic, is reliable and optimal for medium-sized TSP problem (50 to 100 nodes), solves it in short time and finds a good optimum solution. In addition, it is simple easy to use by R&D office workers, and they don't have to learn complex programming concepts.

3. Theoretical model

To solve this problem, I try to break down into 2 different shipments (vehicles) so it will save the time as well and thus increases customer satisfaction and profit. This model is known as multiple TSP. Here, two salesmen must leave the main depot to other different addresses and must return from another different addresses. Otherwise, there should be only one path—in total—taken between cities. The objective function can be formulated as:

$$Minimize\ Z = \sum_{i=1}^{55} \sum_{j=1}^{55} x_{ij} d_{ij}$$

with x_{ij} as the decision variable and d_{ij} as the distance between node i and j.

4. Experiment result and discussion

Table for Vehicle 1

Order no.	Node no.	Distance from prev address (km)
1	1	4.4
2	8	8
3	21	3
4	15	0.4
5	46	4.2
6	25	2.8
7	23	0.9
8	24	6.6
9	45	1.9
10	3	3.3
11	48	1.1
12	39	1.7
13	7	2.1
14	14	3.3
15	50	2
16	18	1.4
17	9	4.3
18	54	3.6
19	52	0
TOTAL		**55.0 km**

Table for Vehicle 2

Order no.	Node no.	Distance from prev address (km)
1	1	6.6
2	20	10.5
3	42	1.3
4	29	1.3
5	22	2.9
6	6	0.3
7	5	2.9
8	19	10.6
9	47	6.8
10	26	6.1
11	36	0.4
12	27	1
13	44	2.1
14	43	0.7
15	37	0.8
16	41	0.7
17	40	1.4
18	31	4.1
19	55	2.7
20	16	3.1
21	51	1.8
22	30	1.6
23	4	1.6
24	13	2.9
25	11	4
26	34	3.9
27	32	4.3
28	17	2.3
29	38	4.5
30	35	0.5
31	33	0.3
32	2	0.6
33	28	0.9
34	53	0.9
35	12	1.4
36	49	2.8
37	10	3.1
TOTAL		**103.7 km**

The solving time using Gurobi Solver Engine, which uses branch-and-bound, is 6.01 seconds—a short time to solve a medium-sized multiple TSP—and can find a good optimum value.

The first vehicle traverses 18 other nodes for 55.0 km before going back to the starting depot, while vehicle 2 traverses 36 other nodes for 103.7 km. The total distance covered by those two vehicles are 158.7 km. It is a very huge distance saving compared to the previous 613.8 km before experiment.

Time-wise, the first vehicle only takes 2.57 hours and the seconds takes 4.85 hours to circulate through Depok, compared to the previous 28.68 hours (which means this task couldn't be completed within one day). This is under the average speed of motor vehicles in Depok of 21.4 km/h.

Fuel-wise, the first vehicle only consumes Rp18,590 and the second consumes Rp35,050.60, with the total consumption of Rp53,640.60. It save a huge Rp153,823.80 compared to the previous Rp207,464,40. The motorcycles that JNE uses consumes one liter each 25 kilometers, with one liter of fuel is equivalent to Rp8,450.

5. Conclusion

In this research, it can be concluded that the Travelling Salesman Problem, especially for the case study of logistics distribution in PT JNE regional Depok, can be solved using the application of linear programming and Solver program efficiently and produce very optimal result and savings in such a short time.

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SL_2-factorisation of twisted Lee-type groups

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The purpose of the investigation

SL_2-factorisation is used in constructing of graphs-expanders, which are necessary to create different networks such as telephone network, computer network, etc.

We are researching Lee-type groups' factorizations as group's decomposition into the product of subgroups which are isomorphic to SL_2 group. These decompositions are well known for Shevalle-type regular groups, but for the twisted Lee-type groups the existing proofs are implicit in nature and give a far from precision upper bound of the factorization's length.

We need to investigate SL_2-factorization of SU_3 group, [1] because of existence of Lee-type groups' decomposition into the product of subgroups, which are isomorphic to SL_2 and SU_3 groups.

For example, the last attempt to assess the length of the factorization was in the article Martin W. Liebeck, Nikolay Nikolov, Aner Shalev *"Groups of Lie type as products of SL_2 subgroups"*, where it is 55, but it is far from precision upper bound of the factorization's length.

Thus, the purpose of our work is giving more precision upper bound of SL_2-factorization's length of SU_3 group.

18 Method of the investigation

19 The research was conducted over a finite field with the dimension equal to a Prime number in even powers of. In order to understand what in fact is happen with the factorization's length of the SU_3 group, we have used a computer program "GAP" (computational mathematics), where we conducted research cases of factorization with different dimensions of the field.

Results of the experiment

In the end we have received and proved more precision upper bound of the factorization's length of SU_3 group in the general case.

Conclusion

As a result, we have a much more accurate upper bound of the factorization's length of SU_3 group in the case when the dimension field is an even power of a Prime. In the future we plan to consider the work in case of arbitrary finite fields.

The Construction of the Derived Functor of the Functor Using the Category of Sheaves

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20 1. Introduction

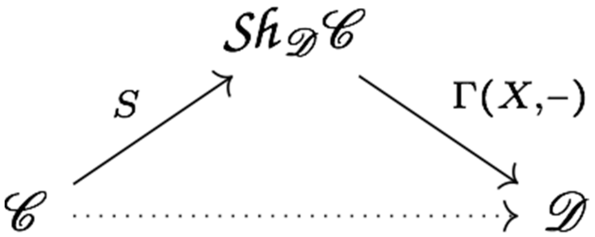
The category theory and homological algebra are the language of modern mathematics and algebra in particular. Last century gave rise not only to these theories, but also to the notion of the derived functor of the functor. Apparently, this notion as well as other ones from the category theory and homological algebra are of considerable importance for modern algebra, physics and theoretical informatics.

The classical definition of the derived functor of the functor suggests that the category-domain has enough projectives, but there are some categories which do not have enough projectives (or there are no projectives at all). The main purpose of this work is to develop a construction which allows us to define the derived functor of the functor without the category-domain having enough projectives.

21 2. Method of the investigations

Alexander Grothedieck is one of the major contributors to higher algebra. In the middle of the last century he introduced a topology on categories – Groethendieck topology. He also developed the sheaf theory and applied it to his new topology. So we can construct presheaves and sheaves on categories. The category of sheaves ("Sh" in the picture) depends on the properties of the category-codomain ("D" in the picture, it is always "good" and has all needed properties, like *Set* or *Ab*) and it does not depend on properties of the original category ("C" in the picture).

Instead of building the derived functor of the original functor (dotted) we introduce a construction which allows us to move all actions to the category of sheaves (using functor "S" in the picture). Then we can use the global sections functor to go to the category-codomain ("D" in the picture).



22 3. Results of the investigation

As a result of our research we have developed a method of constructing the derived functor, which "factors" the original functor through the category of sheaves. Such a construction does not use the property of a category which suggests "having enough projectives/injectives", as long as the classical definition did so.

Two theorems were proven. The first one states that the sheaves on Grothedieck topology are left exact contravariant functors (and inverse). This theorem allows us to regard such functors as sheaves and construct the category of them etc.

The second theorem shows that our construction of the derived functor is equivalent to the classic one, i.e. if we can define the derived functor of the functor, then all the values match and if we cannot, we can do that using our construction.

23 4. Conclusion

The notions that homological algebra and the category theory use are so abstract that all of them can be applied in modern physics and informatics. The notion of the derived functor is one of those and considered to be of paramount importance in homological algebra.

This research generalizes the notion of the derived functor and allows scientists to use it with structures which they could not use before. It means that there is a huge range of unsolved problems in mathematics, physics and other sciences, where the new notion could be applied.

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The Group Commutator Length in Terms of Group Ring

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25 Introduction

26 Let G be a group and [G, G] be its commutator subgroup which is generated by special elements named commutators. Therefore, every element from the commutator subgroup can be represented as a product of commutators. Such representation of an element g from [G, G] that contains the least possible number of commutators is called a minimal commutator representation of g. The number of commutators in a minimal representation of g is called the commutator length of g.

26.1 Method of the investigation

26.2 The purpose of our work is creation of the instrument for transfer the work with commutator length from the group G to its group ring ZG. We give an equivalent definition of commutator length for element g from [G, G], which does not use concept of group commutator, but uses concept of ring commutator in the group ring.

26.3 Results of the experiment

26.4 Now we introduce the following concept. Suppose x, y is from ZG; then the element xy - yx is called the ring commutator of x, y.

26.5 Suppose g is from [G, G]; then there exist a representation of element g - 1 as sum of ring commutators, multiplied on elements from G. The least number of summands in the representation is called ring commutator length of element g - 1. The aim of this paper is to prove the equality commutator length of g and ring commutator length of g - 1.

26.6 Therefore, ring commutator length is equal to the group commutator length. Consequently, it is another definition of the well-known object.

26.7 Conclusion

We believe that the general commutator length theory will greatly benefit from using this definition.

[2]

CONTINUED FRACTIONS AND THE OPTIMIZATION OF HUYGENS' PLANETARIUM MODEL

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1. Introduction

Continued fractions are expressions of the form

$$\alpha = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \ddots}}} \qquad (1)$$

where $a_0 \in Z \wedge a_i \in N (i \geq 1)$

Continued fraction is written in the short form $[a_0; a_1, a_2, \dots, a_{n\dots}]$, citing a sequence of continued fraction digits. For finite continued fraction

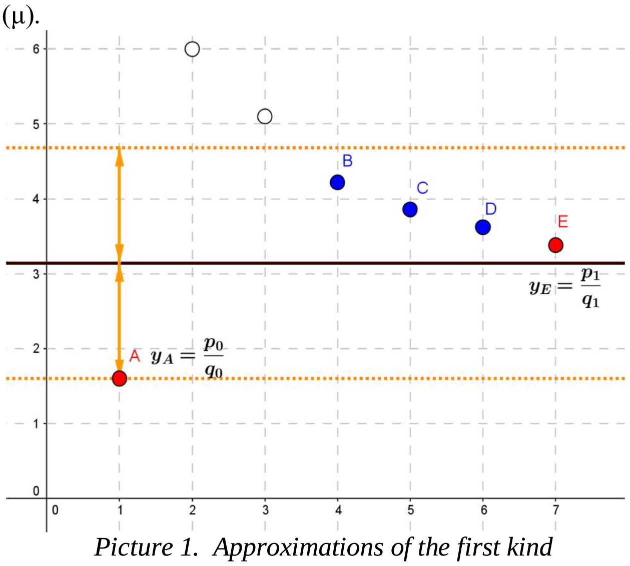
$$\alpha = \frac{p_n}{q_n} = [a_0; a_1, a_2, \dots, a_{n-1,} a_n] \qquad (2)$$

we introduce intermediate fractions

$$\alpha' = \frac{p'_n}{q'_n} = [a_0; a_1, a_2, \dots, a_{n-1}, a'_n] \qquad (3)$$

where $a'_n \in N \wedge 0 < a'_n < a_n$

All continued fractions are in the sequence of approximations of the first kind. We call them continued fraction approximations and mark them with (c). Besides them, in the sequence of approximations of the first kind, there can exist intermediate fractions and we call them intermediate fraction approximations, and mark them with (μ).



Picture 1. Approximations of the first kind

The Huygens' planetarium is a model of orbiting planets based on a system of gears. This provides that the speed at which planets are moving in relation to each other is determined by the ratio of gear teeth of the planets. This connection is found by Huygens through continued fraction approximations, where the 17th century data for orbital period of planets compared to the Earth's were used. Since intermediate fraction approximations provide more precise results than continued fraction approximations, they can be used in the planetarium for finding more accurate gear teeth relations.

The purpose of the project is to introduce Huygens' planetarium model optimization by using continued and intermediate fraction approximations along with NASA's modern day official data of planets' orbital period.

2. Method of the investigation

In order to successfully optimize Huygens' planetarium model an algorithm which finds continued fraction approximations and intermediate fraction approximations was developed and executed in *Python*. This algorithm consists of several parts of process of finding approximations. In the first part it finds continued fraction approximations, then it converts sequences of continued fraction digits to fraction form, so it could be used in the next part for finding intermediate fracitions. In the last section of the algorithm unnecessary checks of approximation quality conditions were avoided [2].

3. Results and discussion

The results obtained with the aforementioned method were compared with Huygens' results and shown in the Table 1. In this table of first smaller and larger approximations of first kind with (c) are marked continued fraction alternatives and with (μ) intermediate fraction alternatives in regard to Huygens' results. By choosing larger alternatives we could get models of planetarium which more correspond to reality.

Table 1 – Huygens' approximations for model of planetarium and found alterantives

	Huygens' approximations	**First smaller and larger approximation of the first kind**
Mercury	$\frac{p_5}{q_5} = \frac{33}{137}$	$\frac{13}{54}(c) < \frac{p_5}{q_5} < \frac{46}{191}(c)$
Venus	$\frac{p_5}{q_5} = \frac{8}{13}$	$\frac{5}{8}(\mu) < \frac{p_5}{q_5} < (\mu)\frac{131}{213}$
Earth	1	1
Mars	$\frac{p_5}{q_5} = \frac{79}{42}$	$\frac{47}{25}(c) < \frac{p_5}{q_5} < \frac{284}{151}(\mu)$
Jupiter	$\frac{p_3}{q_3} = \frac{83}{7}$	$\frac{71}{6}(c) < \frac{p_3}{q_3} < \frac{2395}{202}(\mu)$
Saturn	$\frac{p_1}{q_1} = \frac{59}{2}$	$\frac{29}{1}(c) < \frac{p_1}{q_1} < \frac{147}{5}(c)$

4. Conclusion

In this paper algorithm which finds continued fraction approximations and intermediate approximations was created. It was successfully executed to find the best possible optimizations for Huygens' planetarium model. Obtained results show that by choosing larger alternatives it is able to make calculations for the model of planetarium which more corresponds to reality. Use of continued fractions and intermediate continued fractions is still to be the subject of the future research.

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INVESTIGATION OF REGULAR, SEMI REGULAR AND ISOVALENT TILINGS USING GEOMETRICAL, COMBINATORICAL AND NUMBER THEORY METHODS

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1. INTRODUCTION

Tiling used for making windows, floor covering and wallpapers of parks, palaces and mosque is one of the common areas of interest of science, art and construction. Different tilings shapes are used in work of artist and architect and modelling of capsid virus's geometric forms. In the research papers concerning tilings, there are rather art investigations. In the papers with mathematical approach the polygons which can be used are investigated and some different kinds of tilings were found. In last years these research are concantrated especially to monohedral pentagonal tilings. 15th type of monohedral pentagonal tiling found in 2015, created reaction in science.

Geometrical study of tilings is founded on the research of values of polygon's angles and edges and possible combinations of these angles and edges. The combinatorical approach that is used last 15-20 years is based on Euler's formula on planar graphs. Application of this formula provided some important progress especially on pentagonal tiling's studying.

2. PURPOSE

We developed our hypothesis considering possibilities that some solutions of Diofant Equations that will be obtained in our project will be realised as a tiling and some will be not.

Following this aim in this project we are going to:

- Classify of all regular and semi regular tilings.
- Prove impossibility of tiling by n-gons with $n \geq 7$ which can be found in the literature.
- Classify of tilings by polygons with a same set of valences that is not studied before
- Find cases of realization of this tilings.

Draw the realizable tilings using computer program of GeoGebra.

3. METHOD

To study tilings from mathematical view of point geometrical and combinatorical methods are used. Geometrical study finds all possible angles of polygons and their possible combinations.

Combinatorical approach use Euler's formula for planar graphs. In this project for classification of regular and semi regular tilings, we use geometrical method. To prove that for $n \geq 7$, the plane can not be tiled by convex n-gons we use Euler's formula for planar graphs and lemma of Delone et. al. To classify edge to edge tilings and tilings by polygons, we use the same set of valences, we use also Euler's formula for planar graphs, lemma of Delone et. al. and number theory methods for solving Diofant Equations. To draw realizable tilings and to show no realizability of some solutions of equations as a tiling. We use computer programs GeoGebra and Tessellation Creator.

Combinatorical approach is defined as:

Lemma 1: (Bagina, 2004) Let T be a tiling of the plane by polygons that are uniformly bounded. Then there exists an infinite sequence U_i ($i = 1, 2, 3, \dots$) of finite unions of polygons from T such that

(1) $\tau(U_i) \to \infty$, as $i \to \infty$

(2) $\frac{\gamma(U_i)}{\tau(U_i)} \to 0$, as $i \to \infty$

(3) $\chi(U_i) = 1$ for all i

We prove theorems using this lemma.

Theorem 2: For $n \geq 7$, there is no edge to edge tilings by convex n-gons.

We proved this theorem with classification as edge to edge and non-edge to edge. Then we find this corollary.

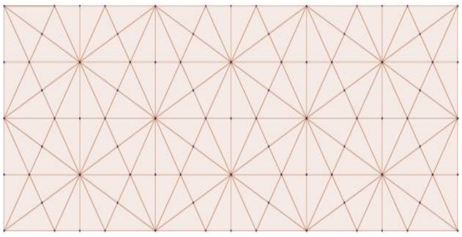
Corollary 3: There are no tilings by convex n-gons for $n \geq 7$

Denote the set of valences odd some n-gon A in tiling is denoted by $D_A = \{d_1, d_2, \dots, d_n\}$ (some numbers d_i in the set D_A may coinside)

Definition: If in a edge to edge tiling K consisting of n-gons the sets D_A of all n-gons D_A are the same then we will say that K an ***isovalent tiling***.

Theorem 4: For the edge to edge tiling consisting of triangles A with $D_A = \{a, b, c\}$, we have

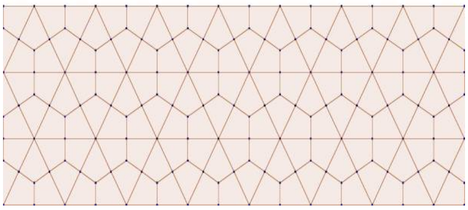
$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{2}$$



One of the solution comes from the equation

Theorem 5: For the edge to edge tiling consisting of quadrilaterals A with $D_A = \{a,b,c,d\}$, we have

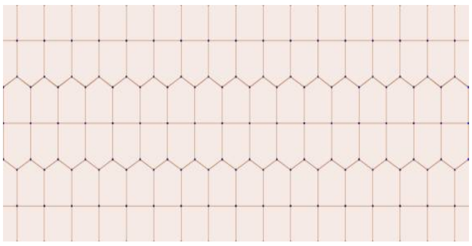
$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d} = 1$$



One of the solution comes from the equation

Theorem 6: For the edge to edge tiling consisting of pentagons A with $D_A = \{a,b,c,d,e\}$, we have

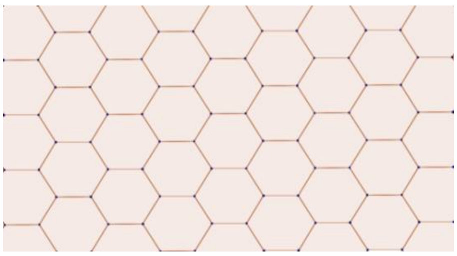
$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}+\frac{1}{e} = \frac{3}{2}$$



One of the solution comes from the equation

Theorem 7: For the edge to edge tiling consisting of hexagons A with $D_A = \{a,b,c,d,e,f\}$, we have

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}+\frac{1}{e}+\frac{1}{f} = 2$$



One of the solution comes from the equation

Geometrical approach is defined as:

To classify all regular and semi regular tilings, we study the polygons which can be combined in a vertex. After the combination of polygons in some vertex the sum of angles in this vertex must be 360°. Since the interior angles of a regular polygon is at least 60°, the number of polygons combined in one vertex is at most 6, this is tiling with equilateral triangles. Since we use convex polygons, the number of polygons combined in vertex at least 3. There are some examples of this case. The simplest of them is tiling with regular hexagons.

If we denote angles combined in one vertex by $x_1, x_2, x_3, x_4, x_5, x_6$ then we have the following equation, which we will call ***main equation***:

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 360^\circ$$

If the number of angles is less than 6, we will assume that the rest valuables x_i are 0.

4. RESULTS AND CONCLUSION

In this Project, we consider tiling of the plane by polygons, investigate important developments including the last ones. We obtained the following results concerning the classification of tilings.

1- We give the classification of tilings by regular polygons with proof. Note that such a classification without proof is given in some research papers.

2- Using Euler's formula for planar graphs, we prove that the plane can not be tiled by n-gons with $n \geq 7$. This result is also given in some papers without proof.

3- Applying Euler's formula to the tilings by polygons with a same set of vertex's valences, we obtained some Diofant Equations.

4- Using number theory methods, we found all positive integer solutions of these Diofant Equations.

5- Using computer programs GeoGebra and Tessellation Creator, we drew tilings obtained from some solutions and proved that other solutions can not be realised as tiling.

By literature search shows that classification of tilings with a same set of vertex's valences is not made before. Giving of definition of isovalent tilings, their classification and the proof of the fact that the plane can not be tiled by n-gons with $n \geq 7$ are the *new (different) sides of the project*.

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A new view on differential games theory, or how to catch the fleeing person

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1 Introduction

1.1 Main idea and aim of the research

Main aim and idea of the research appeared by solving known tasks about chasing and they are exists to find more effective methods of solving tasks about chasing due to transition to reference frame of fleeing person.

Problem of the research is in selecting and math justification of chasing strategy by the help of approaches different from traditional in differential games theory.

1.2 Theoretical part

The tasks about chasing which are met in the collection, where on the first sight the evident strategy of chasing is being realized (S-strategy) when the chaser is moving towards of the fleeing person in any moment of time has been analyzed in this work. When analyzing the comparison of S-strategy was made with the strategy of parallel rapprochement (P-strategy) that was worked out in the differential games theory.

For the simplification of differential games theory in solving tasks about chasing the transition to another reference frame was made. Basing on this L-strategy was worked out, where for any moment of time, in reference frame of fleeing person B the chaser P is moving towards B.

$t^* = \frac{x}{v} = \frac{L}{\sqrt{u^2 - v^2}}$ - time of the persecution in L-strategy

$t_k = \frac{uL}{u^2 - v^2}$ - time of the persecution in S-strategy

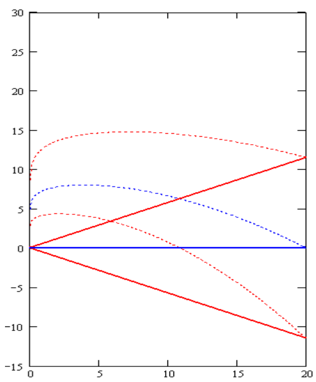
$t^* < t_k$ - time of the persecution in L-strategy is less than in S-strategy.

Thus, it means that L-strategy is more profitable for the chaser.

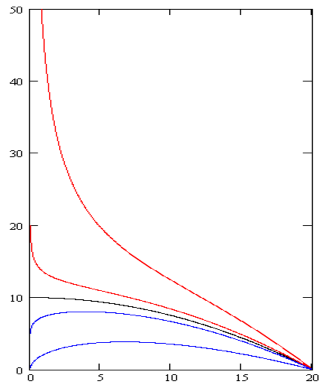
1.3 Experimental part

The experiment was made like computer modeling. The computer modeling of L-strategy clearly proves it's advantages. In the reference frame of fleeing person exactly it becomes clear that L-strategy in the most profitable for chaser.

Computer modeling gives possibilities to compare the players trajectory of movement for different speed ratios of the chaser and the fleeing person and it also allows to get the family of curves that determine the dependence of the distance between the participants of race from time, that makes it possible to watch the features of the relative movement of the chaser and the fleeing person.



A



B

Fig. 1 Computer modeling for analyzing of results:

A – the comparison of movement trajectories of the chaser in B-system for different values of angle between players (dotted lines – movement in S-strategy, solid curves – movement in L-strategy);

B - trajectories of movement of the chaser in B-system depending the ratio of speeds.

2 Results of the experiment

The results of the experimental prove that the suggested L-strategy coincides with the developed in the theory of differential games P-strategy during the transition into the reference frame connected with Earth. The suggested approaches simplify mathematical way of solving and allow easily transfer on 3d or n-d space.

The Computer program that modeling the movement of the chaser and the fleeing person in the reference frame, connected with Earth and allows to imagine the real movement and can be used for the demonstrating the subject of Kinematics.

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The Aquarium Problem: Combinatorical Processes Research

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Introduction

This paper is dedicated to one specific problem in the combinatorial processes filed: complex system develops according to its own laws and at some point meets resource depletion. Our aim is to study the characteristics of this process, first of all, the possible final positions.

Problem definition

We have n aquariums. Each aquarium starts having 1, 2, .., n fish inside respectively. Then we rearrange fish in aquariums according the following rules: we can take two aquariums having both even or both odd (but still different) number of fish and relocate fish to make its number in the aquariums equal. The process continues while we have possible moves. Describe the set of all possible final positions for the process.

Restrictions

We have found two important invariants in the process and the invariants helped us to prove that the process is finite and its maximal length is of cubic dependence on n.

We found two important restrictions for the possible final positions. The first stems from the fact that the number of fish is an integer value, therefore the final positions should be consistent with the number theory.

The second one has unexpected connections in algebra. It is a notion of majorization which is well-known both in matrix theory and inequality studies.

We say sequence x majorizes sequence y if:

a) x and y have the same element number and the same average.

b) for any possible k sum of k largest elements of x is greater than sum of k largest elements of y.

It turned out that in our problem the initial position should majorize all possible final positions.

The main conjecture

We hypothesize that the two restrictions that the final positions have are the only restrictions for the final positions in the problem, i. e. for every position which meet the restrictions there is a chain of steps ending in that position.

Number experiment

We composed a number of assistant computer programs to better understand the process under study. One of them output the full list of possible final positions for given n. Unfortunately, the computational complexity is quickly increasing with increase of n. So we supposed that every possible final position has large enough probability in random process and composed the Monte-Carlo-style program. For the comparison purpose we also composed the program whist enlists possible according to the main conjecture final positions.

Constructions

We have noticed that to get the specific final position it is sufficient to split the numbers 1 to n into two symmetrical (in some sense) sets. In the rest of the article we discuss if it is possible to construct these symmetric halves.

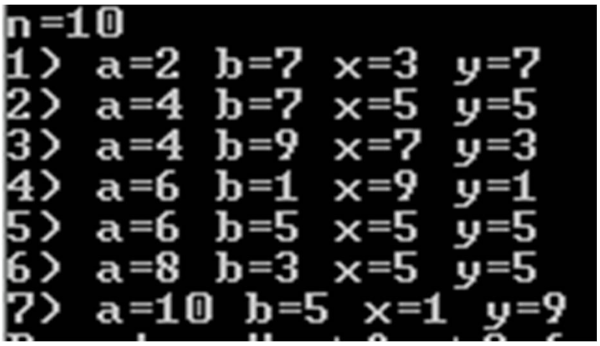

Fig.1. All possible final positions for $n = 10$ (x,y - amount of aquariums, a, b −amount of fish.

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